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MECHANICS.

226. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

A frustum of a cone, vertical angle α, is cut off by two spheres whose centers are the vertex. The radius of one sphere is n times that of the other, and the density of the cone varies as the distance of the vertex. Find the ratio into which the centroid of the frustum divides the axis.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let R=radius of smaller sphere, nR=radius of larger sphere. Take the axis of the cone as that of z. Then $\bar{x}=\bar{y}=0$.

$$\bar{z}=\frac{\int\int\int r^4\sin\theta\cos\theta\,d\theta\,d\phi\,dr}{\int\int\int r^3\sin\theta\,d\theta\,d\phi\,dr}.$$

By frustum of cone here is evidently meant that part of the cone between the spheres.

$$\therefore\bar{z}=\frac{\int_0^{\frac{1}{2}\alpha}\int_0^{2\pi}\int_R^{nR}r^4\sin\theta\cos\theta\,d\theta\,d\phi\,dr}{\int_0^{\frac{1}{2}\alpha}\int_0^{2\pi}\int_R^{Rn}r^3\sin\theta\,d\theta\,d\phi\,dr}=\tfrac{4}{5}R\left(\frac{n^5-1}{n^4-1}\right)\frac{\int_0^{\frac{1}{2}\alpha}\sin\theta\cos\theta\,d\theta}{\int_0^{\frac{1}{2}\alpha}\sin\theta\,d\theta}$$

$$=\tfrac{2}{5}R\left(\frac{n^5-1}{n^4-1}\right)(1+\cos\tfrac{1}{2}\alpha).$$

$$\frac{nR-\bar{z}}{\bar{z}-R}=\frac{5n(n^4-1)-2(n^5-1)(1+\cos\frac{1}{2}\alpha)}{2(n^5-1)(1+\cos\frac{1}{2}\alpha)-5(n^4-1)}.$$

If planes were passed through the intersections of the spheres with the cone, then $R\cos\frac{1}{2}\alpha\,(n-1)$ is the height of the frustum, and the limits of r are $nR\cos\frac{1}{2}\alpha\sec\theta=R_1$, and $R\cos\frac{1}{2}\alpha\sec\theta=R_2$.

$$\therefore\bar{z}=\tfrac{4}{5}R\cos\tfrac{1}{2}\alpha\left(\frac{n^5-1}{n^4-1}\right)\frac{\int_0^{\frac{1}{2}\alpha}\sin\theta\cos\theta\sec^5\theta\,d\theta}{\int_0^{\frac{1}{2}\alpha}\sin\theta\sec^4\theta\,d\theta}=\tfrac{4}{5}\left(\frac{n^5-1}{n^4-1}\right)R\cos\tfrac{1}{2}\alpha.$$

$$\therefore\frac{nR\cos\frac{1}{2}\alpha-\bar{z}}{\bar{z}-R\cos\frac{1}{2}\alpha}=\frac{n^5-5n+4}{4n^5-5n^4+1}.$$

II. Solution by J. A. CAPARO, C. E., Notre Dame University, Notre Dame, Ind.

Let α=one-half the vertical angle of cone, a, and an radii of spheres. With the vertex of the cone as origin, and the axis of y as the axis of the cone, the mass of the frustum contained between the spheres is

$$M=k\pi\tan^2\alpha\int_{a\cos\alpha}^{na\cos\alpha} y^3dy+k\pi\int_{an\cos\alpha}^{an}(a^2-y^2)ydy-k\pi\int_{a\cos\alpha}^{a}(a^2-y^2)ydy$$

$$=\frac{\pi ka^4(n^4-1)\sin^2\alpha}{4}.$$

These integrals represent the mass of a frustum whose bases are planes passing through the line of intersection of the surfaces, and the masses of the spherical segments added and subtracted from the above mass. Call these masses M_c, M_a, M_n, and $\overline{y}_c$, $\overline{y}_a$, $\overline{y}_n$, distances of their centroids from origin.

Then, $M_c\overline{y}_c=\pi k\tan^2\alpha\int_{a\cos\alpha}^{na\cos\alpha} y^4dy=\frac{1}{5}\pi ka^5\cos^5\alpha\tan^2\alpha\,(n^5-1).$

$$M_a\overline{y}_a=\pi k\int_{a\cos\alpha}^{a}(a^2-y^2)y^2dy=\frac{\pi ka^5}{15}(2-5\cos^3\alpha+3\cos^5\alpha).$$

$$M_n\overline{y}_n=\pi k\int_{an\cos\alpha}^{an}(a^2-y^2)y^2dy=\frac{\pi ka^5n^3}{15}[5(1-\cos^3\alpha)-3n^2(1-\cos^5\alpha)].$$

But $M\overline{y}=M_c\overline{y}_c+M_n\overline{y}_n-M_a\overline{y}_a$.

$$\therefore \overline{y}=\frac{4a[2+5n^3-3n^5+n^3\cos^3\alpha(3n^2+5)+2\cos^3(4+3\cos^2\alpha)]}{15(n^4-1)\sin^2\alpha}.$$

227. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Regarding the earth as a homogeneous sphere, radius R, acceleration at the surface g, investigate the motion of a sphere, radius b, moving through a straight tunnel between two points on the surface not diametrically opposite.

Solution by the PROPOSER.

Let $AB=2a$ be the tunnel; $OA=OB=R$; C the mid-point of AB; P and R, points between A and C; $OP=y$, $PC=x$, $CR=d$, $\angle POC=\theta$; f the acceleration at P; f' the acceleration along PC at P, and R the starting point of the sphere. Beneath the earth's surface, the acceleration varies directly as the distance from the center.

$\therefore f:g=y:R$ or $f=gy/R$. $f':f=\sin\theta:1$ or $f'=f\sin\theta=gy\sin\theta/R$.

But $y=x/\sin\theta$. $\therefore f'=gx/R$.